Cambridge Assessment International Education

Cambridge O Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

4139805691

ADDITIONAL MATHEMATICS **4037/13**

Paper 1 **October/November 2022**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

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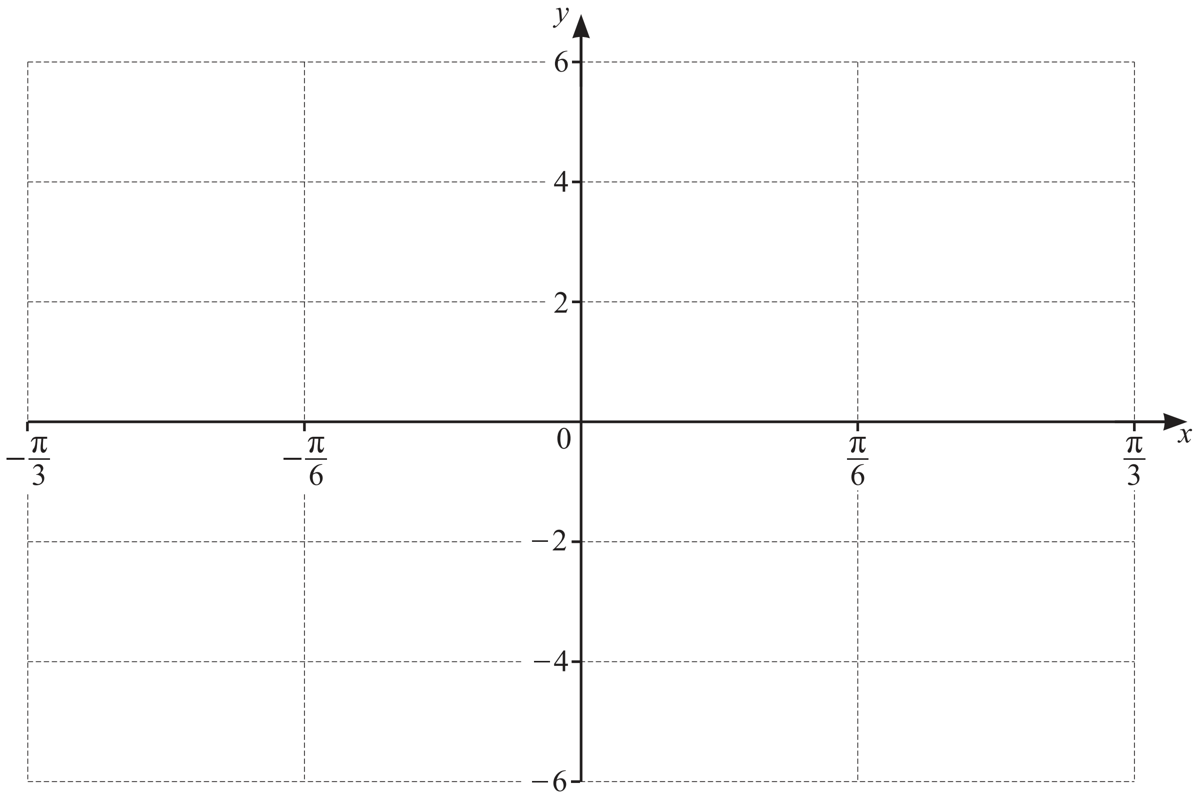
1 On the axes, sketch the graph of $y = 4\sin 3x - 2$ for $-\frac{\pi}{3} \leqslant x \leqslant \frac{\pi}{3}$. [3]



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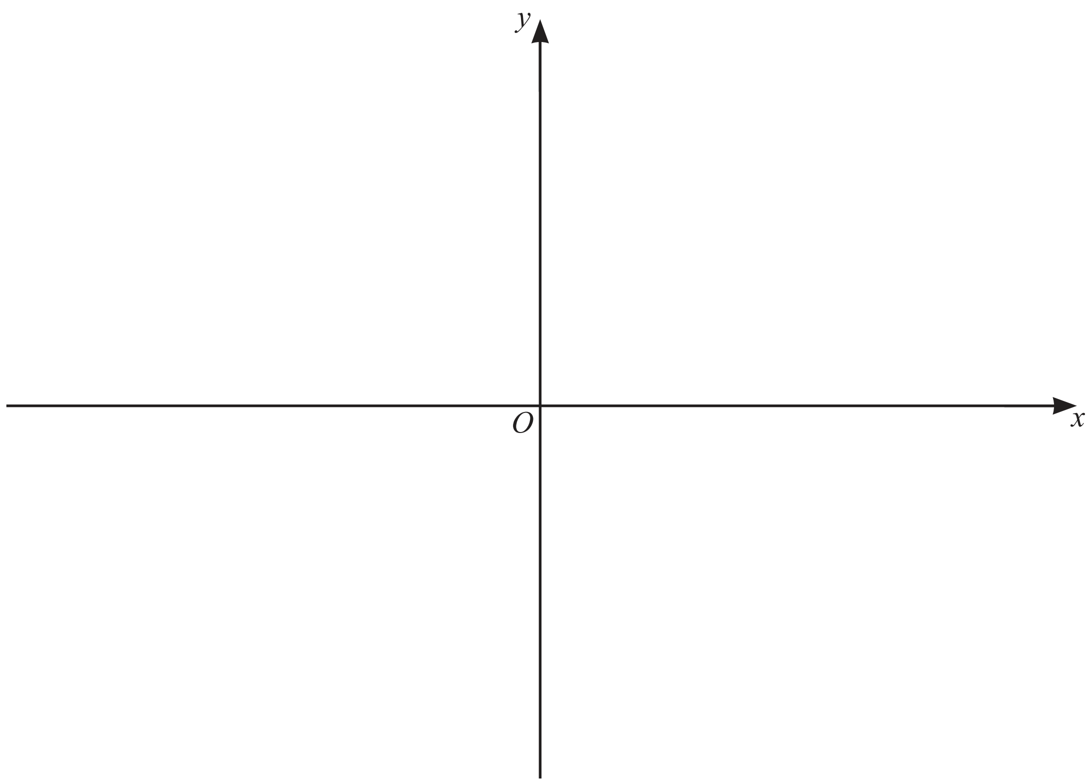
2 **(a)** Show that $2x^2 + x - 15$ can be written in the form $2(x+a)^2 + b$, where a and b are exact constants to be found. [2]

(b) Hence write down the coordinates of the stationary point on the curve $y = 2x^2 + x - 15$. [2]

(c) On the axes, sketch the graph of $y = |2x^2 + x - 15|$, stating the coordinates of the points where the graph meets the coordinate axes. [3]



(d) Write down the value of the constant k for which the equation $|2x^2 + x - 15| = k$ has 3 distinct solutions. [1]

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3 **(a)** Solve the following simultaneous equations.

$$3y - 2x + 2 = 0$$

$$xy = \frac{1}{2}$$ [3]

(b) Solve the equation $\log_3 x + 3 = 10\log_x 3$, giving your answers as powers of 3. [4]

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4 The polynomial $p(x)$ is such that $p(x) = ax^3 + 13x^2 + bx + c$, where a, b and c are integers. It is given that $p'(0) = -9$.

(a) Show that $b = -9$. [1]

It is also given that $3x + 2$ is a factor of $p(x)$ and that when $p(x)$ is divided by $x + 1$ the remainder is 6.

(b) Find the values of a and c. [4]

(c) Find the quadratic $q(x)$ such that $p(x) = (3x + 2) \times q(x)$. [1]

(d) Hence find $p(x)$ as a product of linear factors with integer coefficients. [1]

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5 A geometric progression is such that the fifteenth term is equal to $\frac{1}{8}$ of the twelfth term. The sum to infinity is 5.

(a) Find the first term and the common ratio. [4]

(b) Find the least number of terms needed for the sum of the geometric progression to be greater than 4.999. [3]

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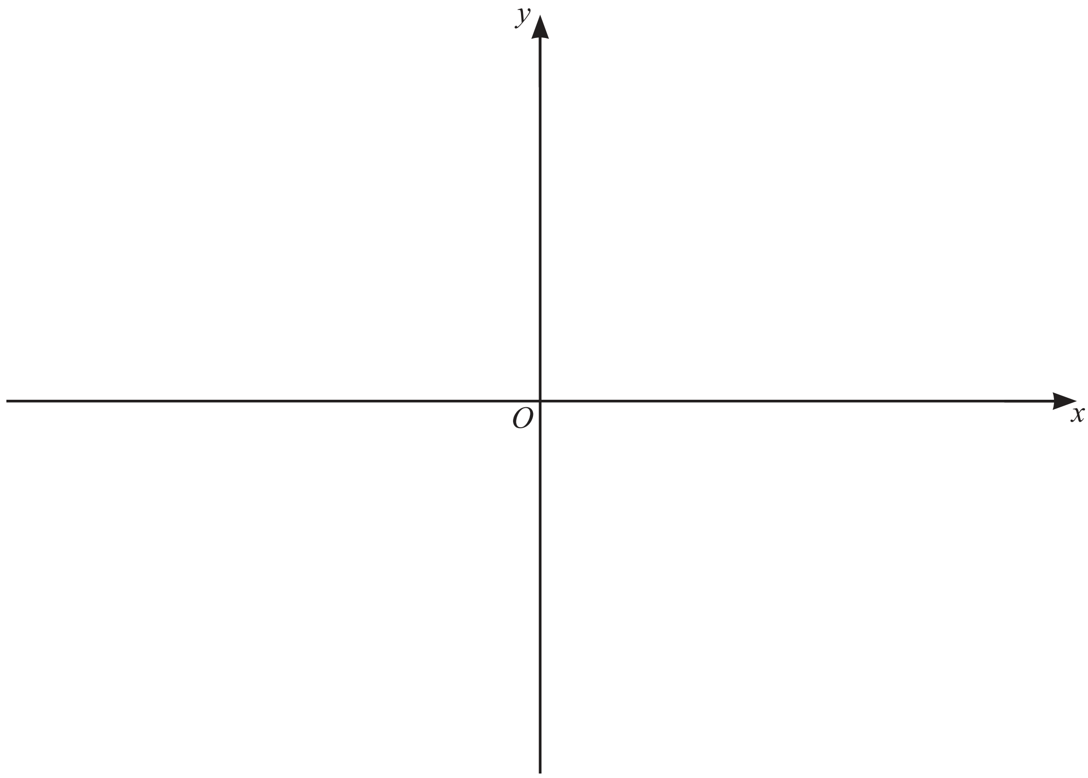
6 A function $f(x)$ is such that $f(x) = e^{3x} - 4$, for $x \in \mathbb{R}$.

(a) Find the range of f. [1]

(b) Find an expression for $f^{-1}(x)$. [2]

(c) On the axes, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ stating the exact values of the intercepts with the coordinate axes. [4]



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7 Find the exact value of $\int_0^{\frac{\pi}{2}} (\cos 3x + 4\sin 2x + 1)\,\mathrm{d}x$. [5]

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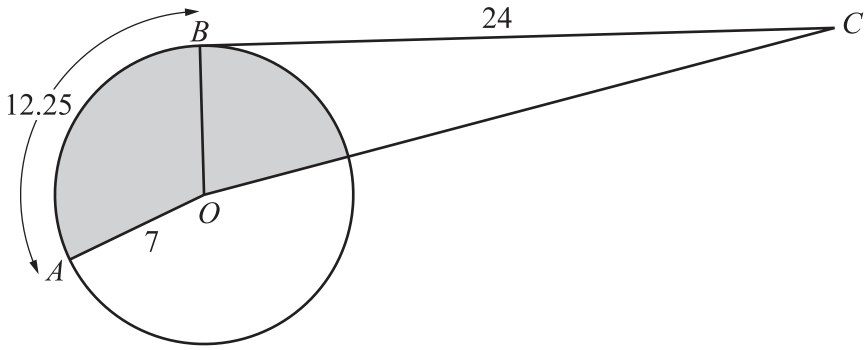
8 In this question all lengths are in metres.



The diagram shows a circle, centre O, radius 7. The points A and B lie on the circumference of the circle. The line BC is a tangent to the circle at the point B such that the length of BC is 24. The length of the minor arc AB is 12.25.

(a) Find the obtuse angle AOB, giving your answer in radians. [1]

(b) Find the perimeter of the shaded region. [4]

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(c) Find the area of the shaded region. [2]

9 A 6-character password is to be formed from the following characters.

Letters	A	B	C	D
Numbers	1	2	3	4
Symbols	*	#	$	£

No character may be used more than once in any password.

(a) **(i)** Find the number of different 6-character passwords that can be formed. [1]

(ii) How many of these 6-character passwords end with a symbol? [1]

(b) Find the number of different 6-character passwords that include all the symbols, but do not start or end with a symbol. [2]

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10 Solve the equation $\sqrt{2}\cos(3x+1.2) = 2\sin(3x+1.2)$, where x is in radians, for $-1.5 \leqslant x \leqslant 1.5$. [5]

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11 It is given that $\int_1^a \left(\frac{3}{3x+2} - \frac{2}{2x+1} - \frac{1}{x}\right) dx = \ln\frac{1}{5}$, where $a > 1$. Find the exact value of a. [6]

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12 It is given that $y = \dfrac{(3x^2-2)^{\frac{2}{3}}}{x-1}$, for $x > 1$.

(a) Write $\dfrac{dy}{dx}$ in the form $\dfrac{(3x^2-2)^{-\frac{1}{3}}}{(x-1)^2}(x^2+Ax+B)$, where A and B are integers. [5]

(b) Find the approximate increase in y as x increases from 2 to $2+p$, where p is small. [2]

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13 The points P and Q have coordinates $(5, -12)$ and $(15, -6)$ respectively. The point R lies on the line l, the perpendicular bisector of the line PQ. The x-coordinate of R is 7.

(a) Find the y-coordinate of R. [4]

(b) The point S lies on l such that its distance from PQ is 3 times the distance of R from PQ. Find the coordinates of the two possible positions of S. [3]

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